In this section, we will solve more equations of the type that we solved in Sections 2.1 and 2.2. However, some equations in this section have infinitely many solutions, and some have no solution.

**Identities**

It is easy to find equations that are satisfied by any real number that we choose as a replacement for the variable. For example, the equations

\[ x + 2 = \frac{1}{2} x, \quad x + x = 2x, \quad \text{and} \quad x + 1 = x + 1 \]

are satisfied by all real numbers. The equation

\[ \frac{5}{x} = \frac{5}{x} \]

is satisfied by any real number except 0 because division by 0 is undefined.

**Identity**

An equation that is satisfied by every real number for which both sides are defined is called an identity.

We cannot recognize that the equation in the next example is an identity until we have simplified each side.

96. **Perimeter of a triangle.** If a triangle has sides of length \( x \), \( x + 1 \), and \( x + 2 \) meters and a perimeter of 12 meters, then the value of \( x \) can be found by solving \( x + (x + 1) + (x + 2) = 12 \). Find the values of \( x \), \( x + 1 \), and \( x + 2 \).

97. **Cost of a car.** Jane paid 9% sales tax and a $150 title and license fee when she bought her new Saturn for a total of $16,009.50. If \( x \) represents the price of the car, then \( x \)

\[ x + 0.09x + 150 = 16,009.50 \]

Find the price of the car by solving the equation.

98. **Cost of labor.** An electrician charged Eunice $29.96 for a service call plus $39.96 per hour for a total of $169.82 for installing her electric dryer. If \( n \) represents the number of hours for labor, then \( n \) satisfies

\[ 39.96n + 29.96 = 169.82 \]

Find \( n \) by solving this equation.
**Example 1**

**Solving an identity**

Solve \(7 - 5(x - 6) + 4 = 3 - 2(x - 5) - 3x + 28\).

**Solution**

We first use the distributive property to remove the parentheses:

\[
\begin{align*}
7 - 5(x - 6) + 4 &= 3 - 2(x - 5) - 3x + 28 \\
7 - 5x + 30 + 4 &= 3 - 2x + 10 - 3x + 28 \\
41 - 5x &= 41 - 5x & \text{Combine like terms.}
\end{align*}
\]

This last equation is true for any value of \(x\) because the two sides are identical. So all real numbers satisfy the original equation, and it is an identity.

---

**Conditional Equations**

The statement \(2x + 4 = 10\) is true only on condition that we choose \(x = 3\). The equation \(x^2 = 4\) is satisfied only if we choose \(x = 2\) or \(x = -2\). These equations are called conditional equations.

**Conditional Equation**

A conditional equation is an equation that is satisfied by at least one real number but is not an identity.

Every equation that we solved in Sections 2.1 and 2.2 is a conditional equation.

---

**Inconsistent Equations**

It is easy to find equations that are false no matter what number we use to replace the variable. Consider the equation

\[x = x + 1.\]

If we replace \(x\) by 3, we get \(3 = 3 + 1\), which is false. If we replace \(x\) by 4, we get \(4 = 4 + 1\), which is also false. Clearly, there is no number that will satisfy \(x = x + 1\). Other examples of equations with no solutions include

\[
\begin{align*}
x &= x - 2, \\
x - x &= 5, \\
0 \cdot x + 6 &= 7.
\end{align*}
\]

**Inconsistent Equation**

An equation that has no solution is called an inconsistent equation.

---

**Example 2**

**Solving an inconsistent equation**

Solve \(2 - 3(x - 4) = 4(x - 7) - 7x\).
Solution

Use the distributive property to remove the parentheses:

\[
\begin{align*}
2 - 3(x - 4) &= 4(x - 7) - 7x & \text{The original equation} \\
2 - 3(x + 12) &= 4x - 28 - 7x & \text{Distributive property} \\
14 - 3x &= -28 - 3x & \text{Combine like terms on each side.} \\
14 - 3x + 3x &= -28 - 3x + 3x & \text{Add } 3x \text{ to each side.} \\
14 &= -28 & \text{Simplify.}
\end{align*}
\]

This last equation is not true for any choice of \(x\). So there is no solution to the original equation, and the equation is inconsistent.

Keep the following points in mind in solving equations.

\[
\text{Summary: Identities and Inconsistent Equations}
\]

1. An equation that is equivalent to an equation in which both sides are identical is an identity. The equation is satisfied by all real numbers for which both sides are defined.
2. An equation that is equivalent to an equation that is always false is inconsistent. The equation has no solution.

Equations Involving Fractions

We solved some equations involving fractions in Sections 2.1 and 2.2. Here, we will solve equations with fractions by eliminating all fractions in the first step. All of the fractions will be eliminated if we multiply each side by the least common denominator.

\section*{Example 3}

Multiplying by the least common denominator

Solve \(\frac{y}{2} - 1 = \frac{y}{3} + 1\)

\section*{Solution}

The least common denominator (LCD) for the denominators 2 and 3 is 6. Since both 2 and 3 divide into 6 evenly, multiplying each side by 6 will eliminate the fractions:

\[
\begin{align*}
6\left(\frac{y}{2} - 1\right) &= 6\left(\frac{y}{3} + 1\right) & \text{Multiply each side by } 6. \\
6 \cdot \frac{y}{2} - 6 \cdot 1 &= 6 \cdot \frac{y}{3} + 6 \cdot 1 & \text{Distributive property} \\
3y - 6 &= 2y + 6 & \text{Simplify: } 6 \cdot \frac{y}{2} = 3y \\
3y &= 2y + 12 & \text{Add } 6 \text{ to each side.} \\
y &= 12 & \text{Subtract } 2y \text{ from each side.}
\end{align*}
\]

Check 12 in the original equation:

\[
\begin{align*}
\frac{12}{2} - 1 &= \frac{12}{3} + 1 \\
6 - 1 &= 4 + 1 \\
5 &= 5
\end{align*}
\]

Since 12 satisfies the original equation, the solution is 12.
Equations involving fractions are usually easier to solve if we first multiply each side by the LCD of the fractions.

**Equations Involving Decimals**

When an equation involves decimal numbers, we can work with the decimal numbers or we can eliminate all of the decimal numbers by multiplying both sides by 10, or 100, or 1000, and so on. Multiplying a decimal number by 10 moves the decimal point one place to the right. Multiplying by 100 moves the decimal point two places to the right, and so on.

**Example 4**

An equation involving decimals

Solve $0.3p + 8.04 = 12.6$.

**Solution**

The largest number of decimal places appearing in the decimal numbers of the equation is two (in the number 8.04). Therefore we multiply each side of the equation by 100 because multiplying by 100 moves decimal points two places to the right:

\[
\begin{align*}
0.3p + 8.04 &= 12.6 & \text{Original equation} \\
100(0.3p + 8.04) &= 100(12.6) & \text{Multiplication property of equality} \\
100(0.3p) + 100(8.04) &= 100(12.6) & \text{Distributive property} \\
30p + 804 &= 1260 & \text{Simplify} \\
30p + 804 - 804 &= 1260 - 804 & \text{Subtract 804 from each side.} \\
30p &= 456 & \\
\frac{30p}{30} &= \frac{456}{30} & \text{Divide each side by 30.} \\
p &= 15.2 & \text{Simplify.}
\end{align*}
\]

You can use a calculator to check that

\[0.3(15.2) + 8.04 = 12.6.\]

The solution is 15.2.

**Example 5**

Another equation with decimals

Solve $0.5x + 0.4(x + 20) = 13.4$.

**Solution**

First use the distributive property to remove the parentheses:

\[
\begin{align*}
0.5x + 0.4(x + 20) &= 13.4 & \text{Original equation} \\
0.5x + 0.4x + 8 &= 13.4 & \text{Distributive property} \\
10(0.5x + 0.4x + 8) &= 10(13.4) & \text{Multiply each side by 10.} \\
5x + 4x + 80 &= 134 & \text{Simplify.} \\
9x + 80 &= 134 & \text{Combine like terms.} \\
9x + 80 - 80 &= 134 - 80 & \text{Subtract 80 from each side.} \\
9x &= 54 & \text{Simplify.} \\
x &= 6 & \text{Divide each side by 9.}
\end{align*}
\]

After you have used one of the properties of equality on each side of an equation, be sure to simplify all expressions as much as possible before using another property of equality. This step is like making sure that all of the injured football players are removed from the field before proceeding to the next play.
Check 6 in the original equation:

\[
0.5(6) + 0.4(6 + 20) = 13.4 \\
3 + 0.4(26) = 13.4 \\
3 + 10.4 = 13.4
\]

Since both sides of the equation have the same value, 6 is the solution.

**CAUTION** If you multiply each side by 10 in Example 5 before using the distributive property, be careful how you handle the terms in parentheses:

\[
10 \cdot 0.5x + 10 \cdot 0.4(x + 20) = 10 \cdot 13.4 \\
5x + 4(x + 20) = 134
\]

It is not correct to multiply 0.4 by 10 and also to multiply \(x + 20\) by 10.

### Simplifying the Process

It is very important to develop the skill of solving equations in a systematic way, writing down every step as we have been doing. As you become more skilled at solving equations, you will probably want to simplify the process a bit. One way to simplify the process is by writing only the result of performing an operation on each side. Another way is to isolate the variable on the side where the variable has the larger coefficient, when the variable occurs on both sides. We use these ideas in the next example and in future examples in this text.

**EXAMPLE 6**

**Simplifying the process**

Solve each equation.

**a)** \(2a - 3 = 0\)  
**b)** \(2k + 5 = 3k + 1\)

**Solution**

**a)** Add 3 to each side, then divide each side by 2:

\[
2a - 3 = 0 \\
2a = 3 \\
a = \frac{3}{2}
\]

Check that \(\frac{3}{2}\) satisfies the original equation. The solution is \(\frac{3}{2}\).

**b)** For this equation we can get a single \(k\) on the right by subtracting \(2k\) from each side. (If we subtract \(3k\) from each side, we get \(-k\), and then we need another step.)

\[
2k + 5 = 3k + 1 \\
5 = k + 1 \\
4 = k
\]

Check that 4 satisfies the original equation. The solution is 4.

**WARM-UPS**

**True or false? Explain your answer.**

1. The equation \(x - x = 99\) has no solution.
2. The equation \(2n + 3n = 5n\) is an identity.
3. The equation \(2y + 3y = 4y\) is inconsistent.
4. All real numbers satisfy the equation $1 \div x = \frac{1}{x}$.
5. The equation $5a + 3 = 0$ is an inconsistent equation.
6. The equation $2t = t$ is a conditional equation.
7. The equation $w - 0.1w = 0.9w$ is an identity.
8. The equation $0.2x + 0.03x = 8$ is equivalent to $20x + 3x = 8$.
9. The equation $\frac{x}{y} = 1$ is an identity.
10. The solution to $3h - 8 = 0$ is $\frac{8}{3}$.

4. Solve each equation. Identify each as a conditional equation, an inconsistent equation, or an identity. See Examples 1 and 2.

7. $x + x = 2x$
8. $2x - x = x$
9. $a - 1 = a + 1$
10. $r + 7 = r$
11. $3y + 4y = 12y$
12. $9t - 8t = 7$
13. $-4 + 3(w - 1) = w + 2(w - 2) - 1$
14. $4 - 5(w + 2) = 2(w - 1) - 7w - 4$
15. $3(m + 1) = 3(m + 3)$
16. $5(m - 1) - 6(m + 3) = 4 - m$
17. $x + x = 2$
18. $3x - 5 = 0$
19. $2 - 3(5 - x) = 3x$
20. $3 - 3(5 - x) = 0$
21. $(3 - 3)(5 - z) = 0$
22. $(2 \cdot 4 - 8)p = 0$
23. $\frac{0}{x} = 0$
24. $\frac{2x}{2} = x$
25. $x \cdot x = x^2$
26. $\frac{2x}{2x} = 1$

Solve each equation by first eliminating the fractions. See Example 3.

27. $\frac{x}{2} + 3 = x - \frac{1}{2}$
28. $13 - \frac{x}{2} = x - \frac{1}{2}$
29. $\frac{x}{2} + \frac{x}{3} = 20$
30. $\frac{x}{2} - \frac{x}{3} = 5$
31. $\frac{w}{2} + \frac{w}{4} = 12$
32. $\frac{a}{4} - \frac{a}{2} = -5$
33. $\frac{3z}{2} - \frac{2z}{3} = -10$
34. $\frac{3m}{4} + \frac{m}{2} = -5$
35. $\frac{1}{5}p - 5 = \frac{1}{4}p$
36. $\frac{1}{2}q - 6 = \frac{1}{5}q$
37. $\frac{1}{6}v + 1 = \frac{1}{4}v - 1$
38. $\frac{1}{15}k + 5 = \frac{1}{6}k - 10$

Solve each equation by first eliminating the decimal numbers. See Examples 4 and 5.

39. $x - 0.2x = 72$
40. $x - 0.1x = 63$
41. $0.3x + 1.2 = 0.5x$
42. $0.4x - 1.6 = 0.6x$
43. $0.02x - 1.56 = 0.8x$
44. $0.6x + 10.4 = 0.08x$
45. $0.1a - 0.3 = 0.2a - 8.3$
46. \(0.5b + 3.4 = 0.2b + 12.4\)
47. \(0.05r + 0.4r = 27\)
48. \(0.08t + 28.3 = 0.5t - 9.5\)
49. \(0.05y + 0.03(y + 50) = 17.5\)
50. \(0.07y + 0.08(y - 100) = 44.5\)
51. \(0.1x + 0.05(x - 300) = 105\)
52. \(0.2x - 0.05(x - 100) = 35\)

Solve each equation. If you feel proficient enough, try simplifying the process, as described in Example 6.

53. \(2x - 9 = 0\)
54. \(3x + 7 = 0\)
55. \(-2x + 6 = 0\)
56. \(-3x - 12 = 0\)
57. \(\frac{x}{5} + 1 = 6\)
58. \(\frac{s}{2} + 2 = 5\)
59. \(\frac{c}{2} - 3 = -4\)
60. \(\frac{b}{3} - 4 = -7\)
61. \(3 = t + 6\)
62. \(-5 = y - 9\)
63. \(5 + 2q = 3q\)
64. \(-4 - 5p = -4p\)
65. \(8x - 1 = 9 + 9x\)
66. \(4x - 2 = -8 + 5x\)
67. \(-3x + 1 = -1 - 2x\)
68. \(-6x + 3 = -7 - 5x\)

Solve each equation.

69. \(3x - 5 = 2x - 9\)
70. \(5x - 9 = x - 4\)
71. \(x + 2(x + 4) = 3(x + 3) - 1\)
72. \(u + 3(u - 4) = 4(u - 5)\)
73. \(23 - 5(3 - n) = -4(n - 2) + 9n\)
74. \(-3 - 4(t - 5) = -2(t + 3) + 11\)
75. \(0.05x + 30 = 0.4x - 5\)
76. \(x - 0.08x = 460\)
77. \(\frac{-2a + 1}{3} = 2\)
78. \(\frac{-3}{4} = \frac{1}{2}\)
79. \(\frac{y}{2} + \frac{y}{6} = 20\)
80. \(\frac{3w}{5} - 1 = \frac{w}{2} + 1\)
81. \(0.09x - 0.2(x + 4) = -1.46\)
82. \(0.08x + 0.5(x + 100) = 73.2\)
83. \(436x - 789 = -571\)
84. \(0.08x + 4533 = 10x + 69\)
85. \(\frac{x}{344} + 235 = 292\)
86. \(34(x - 98) = \frac{x}{2} + 475\)

Solve each problem.

87. **Sales commission.** Danielle sold her house through an agent who charged 8% of the selling price. After the commission was paid, Danielle received $117,760. If \(x\) is the selling price, then \(x\) satisfies

\[x - 0.08x = 117,760.\]

Solve this equation to find the selling price.

88. **Raising rabbits.** Before Roland sold two female rabbits, half of his rabbits were female. After the sale, only one-third of his rabbits were female. If \(x\) represents his original number of rabbits, then

\[\frac{1}{2}x - 2 = \frac{1}{3}(x - 2).\]

Solve this equation to find the number of rabbits that he had before the sale.

89. **Eavesdropping.** Reginald overheard his boss complaining that she federal income tax for 1997 was $34,276.

a) Use the accompanying graph to estimate his boss’s taxable income for 1997.

b) Find his boss’s exact taxable income for 1997 by solving the equation

\[22,532 + 0.31(x - 99,600) = 34,276.\]

**FIGURE FOR EXERCISE 89**

90. **Federal taxes.** According to Bruce Harrell, CPA, the federal income tax for a class C corporation is found by solving a linear equation. The reason for the equation is that the amount \(x\) of federal tax is deducted before the state tax is figured, and the amount of state tax is deducted before the federal tax is figured. To find the amount of federal tax for a corporation with a taxable income of $200,000, for which the federal tax rate is 25% and the state tax rate is 10%, Bruce must solve

\[x = 0.25[200,000 - 0.10(200,000 - x)].\]

Solve the equation for Bruce.